Height-Diameter Equations for Ten Tree Species in the Inland Northwest

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ABSTRACT. *Individual tree height-diameter equations were developed for ten major species in the inland Northwest. The Wykoff function in the Stand Prognosis Model and the Lundqvist function were fit to data which included many large-sized trees. The two models fit the data equally well for all species. Prediction results using the existing Prognosis equation, the refitted Wykoff function, and the Lundqvist function showed that the three models predicted similar heights for trees of small diameter. However, both the refitted Wykoff function and the Lundqvist function predicted larger tree heights for trees with dbh greater than 20 in. for most species. The estimated heights for tree diameters of 70 or 80 in. from the Lundqvist function were closer to the observed "asymptotic" tree heights than the other two. The Lundqvist function showed lower prediction errors for the validation data for the majority of the tree species, especially for large-sized trees. West. J. Appl. For. 11(4):132–137.*

Individual tree height and diameter are essential variables in forest inventory and growth and yield modeling. In practice, tree diameter measurements can easily be obtained at low cost. Tree height measurements, however, are considerably more difficult and costly to collect. To date, many tree height-diameter equations have been developed for various tree species (e.g., Curtis 1967, Huang et al. 1992, Wykoff et al. 1982). When tree height measurements are missing, these prediction models can be applied to estimate the heights using the observed tree diameters. Estimates of other tree characters such as tree volume are dependent on the accurate measurements or predictions for tree diameter and height. Although most of the published height-diameter equations fit the available data adequately and behave well within the range of the model development data, some of the models have been found to underpredict tree heights for large-sized trees. The problem results either from model formulations or height-diameter data collected only from smaller trees (Zhang et al. 1995). Thus, the models may produce large extrapolation errors for large-sized trees.

The objective of this study was to develop new tree height-diameter equations for the ten major tree species in the inland Northwest. These model development data represent a wide range of site productivities, tree ages, and tree sizes (especially large-sized trees).

Data and Methods

We used 47,838 trees of ten species with observed diameter at breast height (dbh, to the nearest 0.1 inch) outside bark and total height (ht, to the nearest foot for large trees and to the nearest 0.1 ft for smaller trees) to develop and test height-diameter equations. The study area covers western Montana, northeastern Oregon, and eastern Washington, but most data were from northern Idaho. The ten tree species are: (1) Western white pine *(Pinus monticola)*, (2) Western larch *(Larix occidentalis)*, (3) Douglas-fir (*Pseudotsuga menziesii* var. *glauca*), (4) Grand fir *(Abies grandis)*, (5) Western hemlock *(Tsuga heterophylla)*, (6) Western redcedar *(Thuja plicata)*, (7) Lodgepole pine *(Pinus contorta)*, (8) Engelmann spruce *(Picea engelmannii)*, (9) Subalpine fir *(Abies lasiocarpa)*, and (10) Ponderosa pine *(Pinus ponderosa)*. The available tree height-diameter data were divided into two sets. The majority of the data (90%) were used for model development. Ten percent of the trees were systematically selected across the range of diameters for each species and reserved for model validation. Summary statistics of tree dbh and ht by species for both data sets are provided in Tables 1 and 2.

Table 1 Summary statistics of tree height (ht) and diameter (dbh for the model development data and the big tree records for ten tree species in the inland Northwest.

Species	No. of trees	Dbh (in.)			HT (ft)			Big Tree	
		Mean	Min.	Max.	Mean	Min.	Max.	Dbh	HT
Western white pine	829	16.3	0.10	58.1	88.1	4.5	185	81.8	214
Western larch	4626	7.1	0.10	76.2	42.8	4.5	206	93.3	175
Douglas-fir	16751	12.7	0.10	53.3	70.0	4.9	200	70.3	209
Grand fir	9466	19.3	0.10	63.5	92.7	4.5	239	70.0	181
Western hemlock	1068	22.2	0.10	54.5	93.7	4.8	160	58.0	165
Western redcedar	6121	30.0	0.10	99.5	103.1	5.0	200	216.6	177
Lodgepole pine	694	9.7	0.10	32.7	57.1	4.5	121	43.6	135
Engelmann spruce	1323	14.1	0.10	42.4	64.2	4.5	164	92.4	179
Subalpine fir	463	14.3	0.10	42.7	63.1	4.5	136	49.9	137
Ponderosa pine	1856	17.4	0.20	87.4	80.0	4.8	195	72.6	172

Many nonlinear equations have been used for modeling tree height-diameter relationships (e.g., Huang et al. 1992). In this study, the following two functions were selected as candidate height-diameter equations:

1. Wykoff function (Wykoff et al. 1982):

$$HT = 4.5 + e^{\left(a + \frac{b}{DBH+1}\right)}$$

2. Lundqvist function (Stage 1963, Zeide 1989):

$$HT = 4.5 + a * e^{(-b*DBH^{-c})}$$

The Wykoff function is used in the Stand Prognosis Model (Wykoff et al. 1982) which has been widely applied in the inland Northwest. We refit this function to our data. The Lundqvist function (some authors call it the Korf function) showed good prediction performance according to previous studies (Zeide 1989, Zhang et al. 1995).

The two functions were fit to the data for each species using PROC NLIN procedure in Statistical Analysis System (SAS Institute Inc. 1990). The validity of least-squares assumptions was investigated. Unequal error variances were not found. This had been a problem in other studies (e.g., Huang et al. 1992). Thus, ordinary nonlinear least-squares was utilized for parameter estimations. Multiple initial values were used to ensure that the least-squares solution was the global rather than the local solution. The squared root of mean squared error of the model was examined for each equation. Simulations using the two equations were compared with the existing equations in the Stand Prognosis Model (Wykoff et al. 1982) for each species. The predicted "asymptotic heights" were also compared with the recorded heights (Table 1) of the champion big trees for the ten species within the study area (Mahoney 1993). This comparison was considered a proper justification for evaluating growth functions (Zeide 1989). The model validation data were divided into three classes: < 15 in., 15–30 in., and > 30 in. There were usually more than ten trees in each class. The existing Prognosis equation, the refitted Wykoff equation, and the Lundqvist equation were used to predict tree heights for each species. Prediction error is defined as the difference between the observed tree height and the predicted tree height. Positive prediction error means underprediction, and negative error indicates overprediction.

Results and Discussion

The Wykoff and Lundqvist functions were fit to tree height-diameter data for each species. The parameter esti-

Table 2. Summary statistics of tree height (ht) and diameter (dbh) for the model validation data for ten tree species in the inland Northwest.

Species	No. of trees	Dbh (in.)			Ht (ft)		
		Mean	Min.	Max.	Mean	Min.	Max.
Western white pine	51	16.6	0.20	48.0	92.3	5.3	182
Western larch	514	7.2	0.10	87.4	43.2	4.9	182
Douglas-fir	1861	12.7	0.10	52.3	70.1	5.0	162
Grand fir	1051	19.3	0.10	52.8	93.0	4.6	189
Western hemlock	118	22.2	0.10	47.0	94.0	5.1	150
Western redcedar	680	30.1	0.26	99.3	103.2	18.1	180
Lodgepole pine	77	9.8	0.30	26.2	59.5	6.8	121
Engelmann spruce	147	14.3	0.10	53.7	64.0	4.6	155
Subalpine fir	51	14.4	0.20	34.3	62.8	5.0	123
Ponderosa pine	190	17.3	0.30	48.3	80.4	6.0	169

Table 3 Parameter estimates and square root of the mean squared error *(RMSE)* of the two equations for ten tree species in the inland Northwest.

Species	Wykoff function			Lundqvist function			
	a	*b*	*RMSE*	*a*	*b*	*c*	*RMSE*
Western white pine	5.3004	−12.4386	14.8	267.40	6.2339	0.6493	14.7
Western larch	5.2554	−11.2760	10.6	1286.04	5.4879	0.2623	9.9
Douglas-fir	5.0207	−9.8882	11.6	995.29	4.9996	0.2490	10.9
Grand fir	5.1472	−10.3828	14.5	314.37	4.7162	0.4783	14.2
Western hemlock	4.9079	−8.0119	18.6	132.12	7.1246	1.0019	18.6
Western redcedar	5.1039	−14.5886	15.9	186.37	7.7294	0.7520	15.9
Lodgepole pine	4.7979	−7.5469	11.9	120.21	5.4930	0.9132	12.0
Engelmann spruce	5.1282	−12.7680	11.8	559.23	4.9596	0.3337	11.6
Subalpine fir	4.9180	−9.8747	11.4	172.15	5.2187	0.6674	11.5
Ponderosa pine	5.1094	−11.9354	14.9	1769.17	5.7742	0.2197	13.8

mates and square root of the mean squared error (RMSE) for each model by species are provided in Table 3. The two functions fit the data equally well. The model RMSEs ranged from 11 ft (western larch) to 19 ft (western hemlock). The estimated parameters for the Wykoff function were slightly different from those in the Stand Prognosis Model (Wykoff et al. 1982, Table 9 on p. 52).

Estimated heights were obtained using the three equations for each species. Tree diameter was set within the range of the model development data (Table 1). Simulation results are shown in Figure 1 for each species. In general, the three equations produced similar tree heights for small trees. For example, the estimated tree height for a 10 in. grand fir tree was 75 ft from the existing Prognosis equation, 71 ft from the refitted Wykoff function, and 70 ft from the Lundqvist function. However, the refitted Wykoff function predicted larger tree heights for tree DBH larger than 20 inches for all species, except western
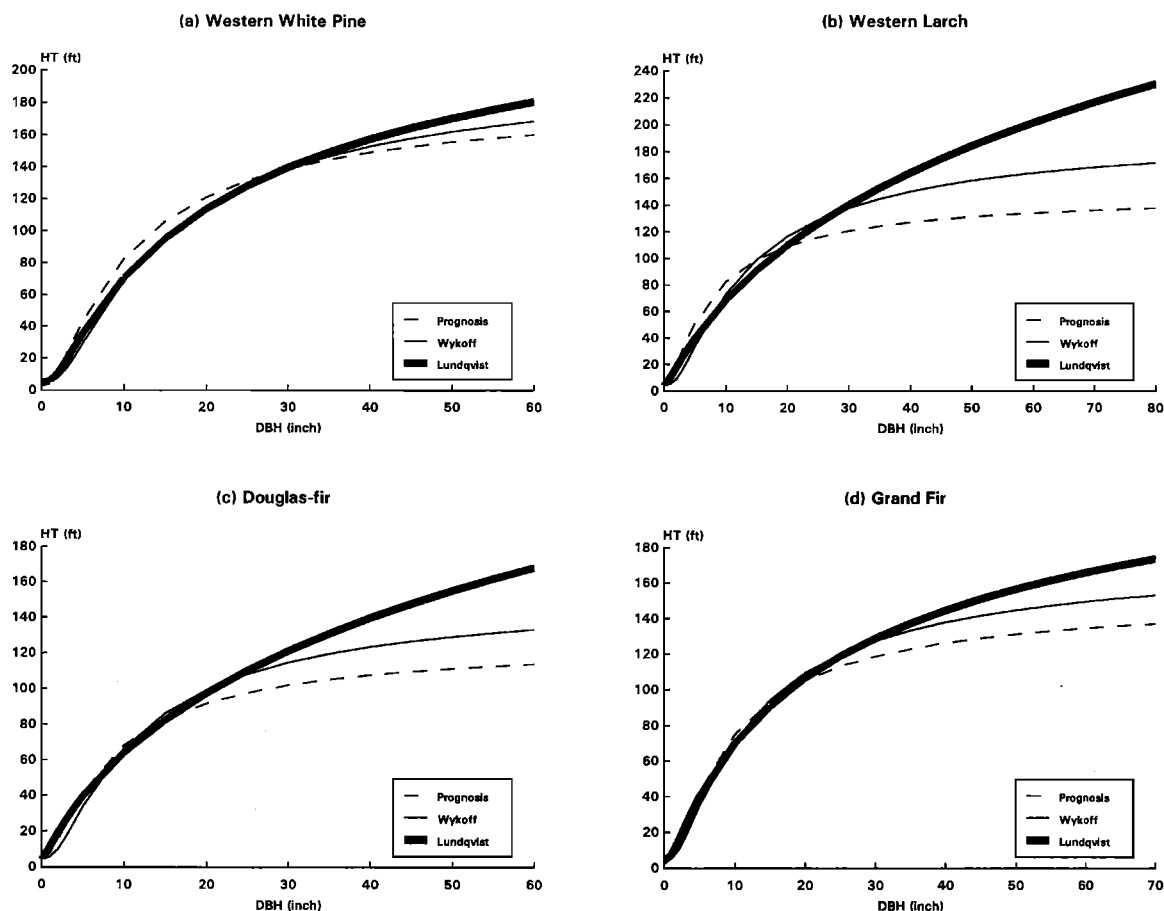
Figure 1. Comparison of the predictions from the tree height-diameter equations in the Stand Prognosis Model, the refitted Wykoff function, and the Lundqvist function for the ten species: (a) western white pine, (b) western larch, (c) Douglas-fir, (d) grand fir, (e) western hemlock, (f) western redcedar, (g) lodgepole pine, (h) Engelmann spruce, (i) subalpine fir, and (j) ponderosa pine. **(cont.)**

(e) Western Hemlock

(f) Western Redcedar

(g) Lodgepole Pine

(h) Engelmann Spruce

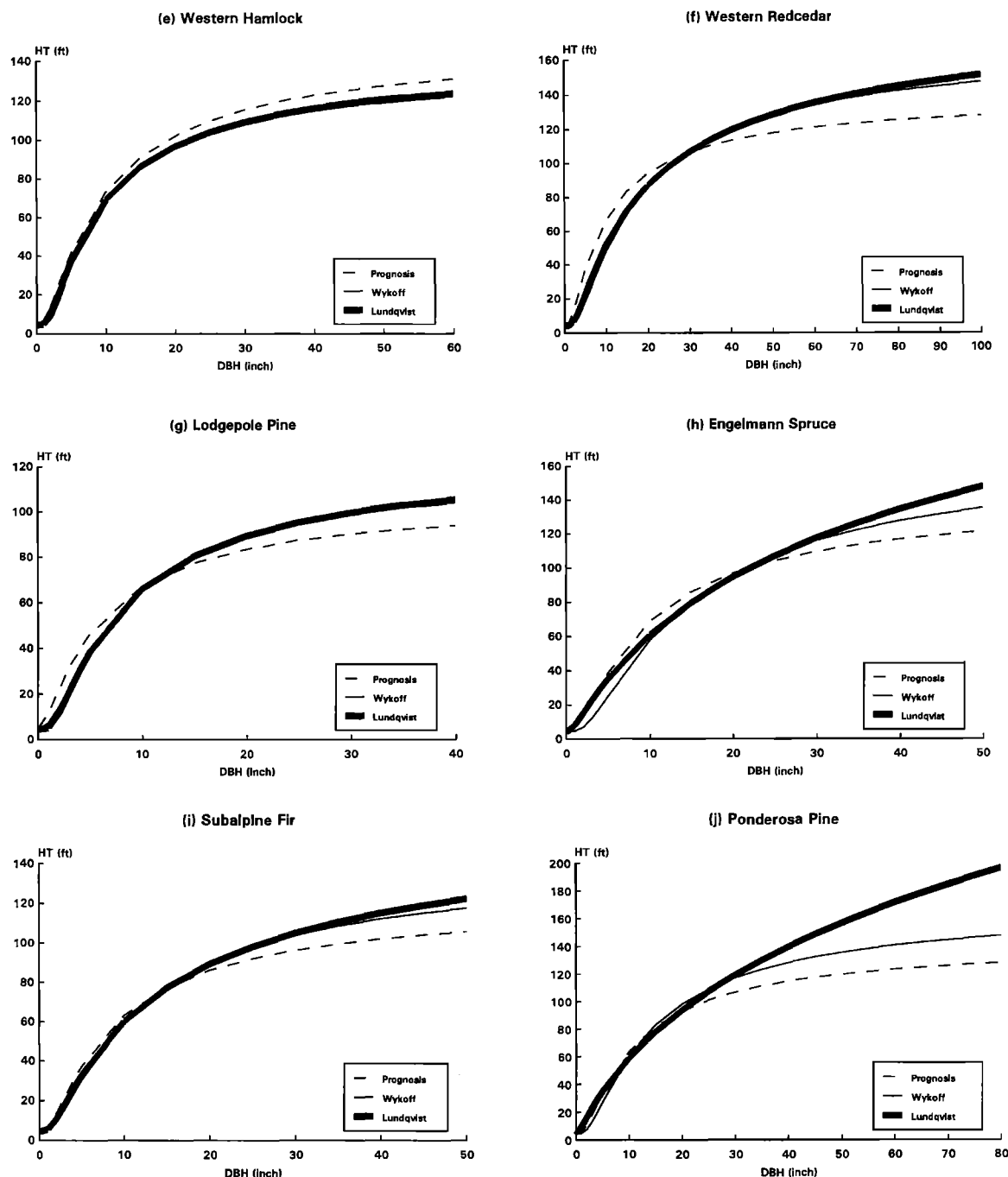**(i) Subalpine Fir**

(j) Ponderosa Pine

Figure 1 (cont.)

hemlock (Figure 1(e)). The Lundqvist function produced even larger estimation of tree heights for large-sized trees and were closer to the observed champion big tree heights. For example, the estimated tree height for a 70 in. grand fir tree was 137 ft from the existing Prognosis equation, 153 ft from the refitted Wykoff function, and 174 ft from the Lundqvist function. The observed grand fir champion big tree had 70 in. of diameter and 181 ft of height.

The average prediction error for each dbh class and overall mean prediction error across the dbh classes are illustrated for each equation and species (Figure 2). If the number of validation trees was two trees or less for any dbh class, the average prediction error for that class was not computed (e.g., Figure 2(i)). In general, the three equations produced similar prediction errors for small trees. For large-sized trees, however, the existing Prognosis equations generated larger underprediction of tree heights for most of the species, except for western hemlock [Figure 2(e)]. The Lundqvist function performed better for the majority of the tree species.

Consequences of Differences in Predicted Height

Tree volume is commonly expressed as a function of tree diameter and total height. To investigate the implication of

(a) Western White Pine   (b) Western Larch
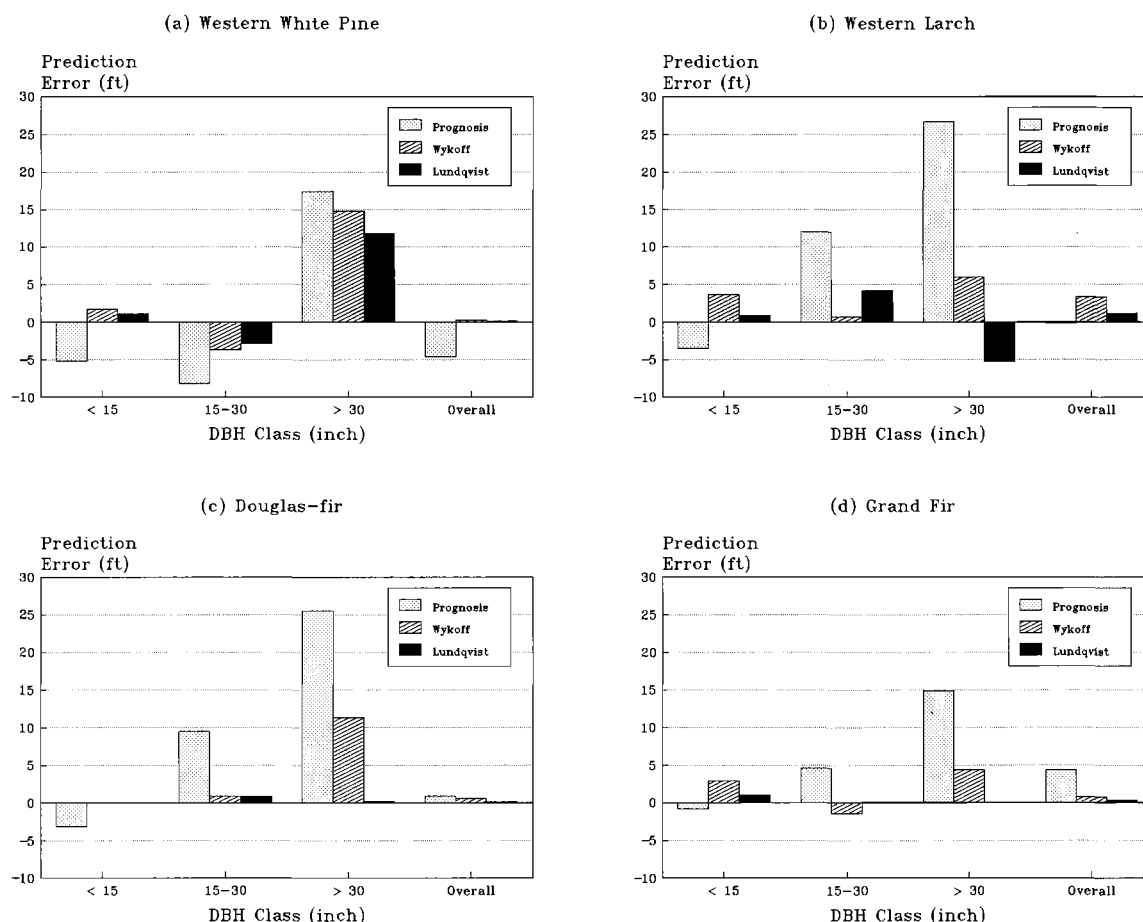
(c) Douglas-fir   (d) Grand Fir

Figure 2. Prediction error from the tree height-diameter equations in the Stand Prognosis Model, the refitted Wykoff function, and the Lundqvist function for the ten species: (a) western white pine, (b) western larch, (c) Douglas-fir, (d) grand fir, (e) western hemlock, (f) western redcedar, (g) lodgepole pine, (h) Engelmann spruce, (i) subalpine fir, and (j) ponderosa pine. (cont.)

tree total height prediction on volume estimation, we set tree diameters at 10, 20, 30, 40, and 50 in., and predicted the corresponding tree total heights using the refitted Wykoff function and the Lundqvist function for each species. Then the individual tree volume equations in the Stand Prognosis Model for the ten species (Wykoff et al. 1982, Table 20 on p. 81) were used to compute the tree volume (Table 4). The results showed that similar tree volume estimations were obtained for trees less than 30 in. dbh for all species. For large-sized trees (dbh > 30 in.), however, the tree heights and therefore volume estimates based on the Lundqvist function

were larger than those based on the refitted Wykoff function for seven of the ten species (except western hemlock, western redcedar, and lodgepole pine). For example, the tree volume estimation based on the Lundqvist function was on average 6.8% larger for dbh = 40 in. and 11.4% larger for dbh = 50 in., respectively, than the estimation based on the refitted Wykoff function.

For trees less than 30 in., either equation can be used However, for inventories or simulations containing large trees, we recommend the Lundqvist function due to its theoretical properties (Ziede 1993).

Table 4. Estimates of tree volume (ft³/tree) based on five tree diameters (10, 20, 30, 40 and 50 in.) and the corresponding predictions of tree total height from the two equations for ten tree species in the inland Northwest.

Species	Wykoff function					Lundqvist function				
	10"	20"	30"	40"	50"	10"	20"	30"	40"	50"
Western white pine	16.1	107.5	290.8	568.4	941.0	16.4	106.4	292.1	581.5	979.0
Western larch	13.5	85.7	228.0	441.7	727.2	12.6	80.9	232.1	483.5	848.0
Douglas-fir	13.9	75.5	189.7	357.1	577.8	13.4	74.1	200.5	404.0	693.4
Grand fir	16.7	102.4	268.6	516.6	847.0	16.4	99.7	271.5	541.6	916.2
Western hemlock	15.3	84.9	214.9	405.8	658.0	15.2	85.2	214.5	403.7	652.6
Western redcedar	9.9	71.1	198.1	393.2	657.0	10.6	71.5	197.2	392.1	658.3
Lodgepole pine	17.9	92.6	225.0	413.0	655.0	18.0	92.1	222.5	407.4	645.2
Engelmann spruce	12.0	73.3	192.4	370.1	607.0	12.7	72.0	195.3	392.7	671.4
Subalpine fir	12.5	68.4	172.1	324.1	524.5	12.6	68.0	173.7	332.4	545.6
Ponderosa pine	13.4	95.6	258.9	505.3	835.3	13.0	90.8	263.7	552.8	974.3

(e) Western Hemlock

Prediction
Error (ft)

(f) Western Redcedar

Prediction
Error (ft)

(g) Lodgepole Pine

Prediction
Error (ft)

(h) Englemann Spruce

Prediction
Error (ft)

(i) Subalpine Fir

Prediction
Error (ft)

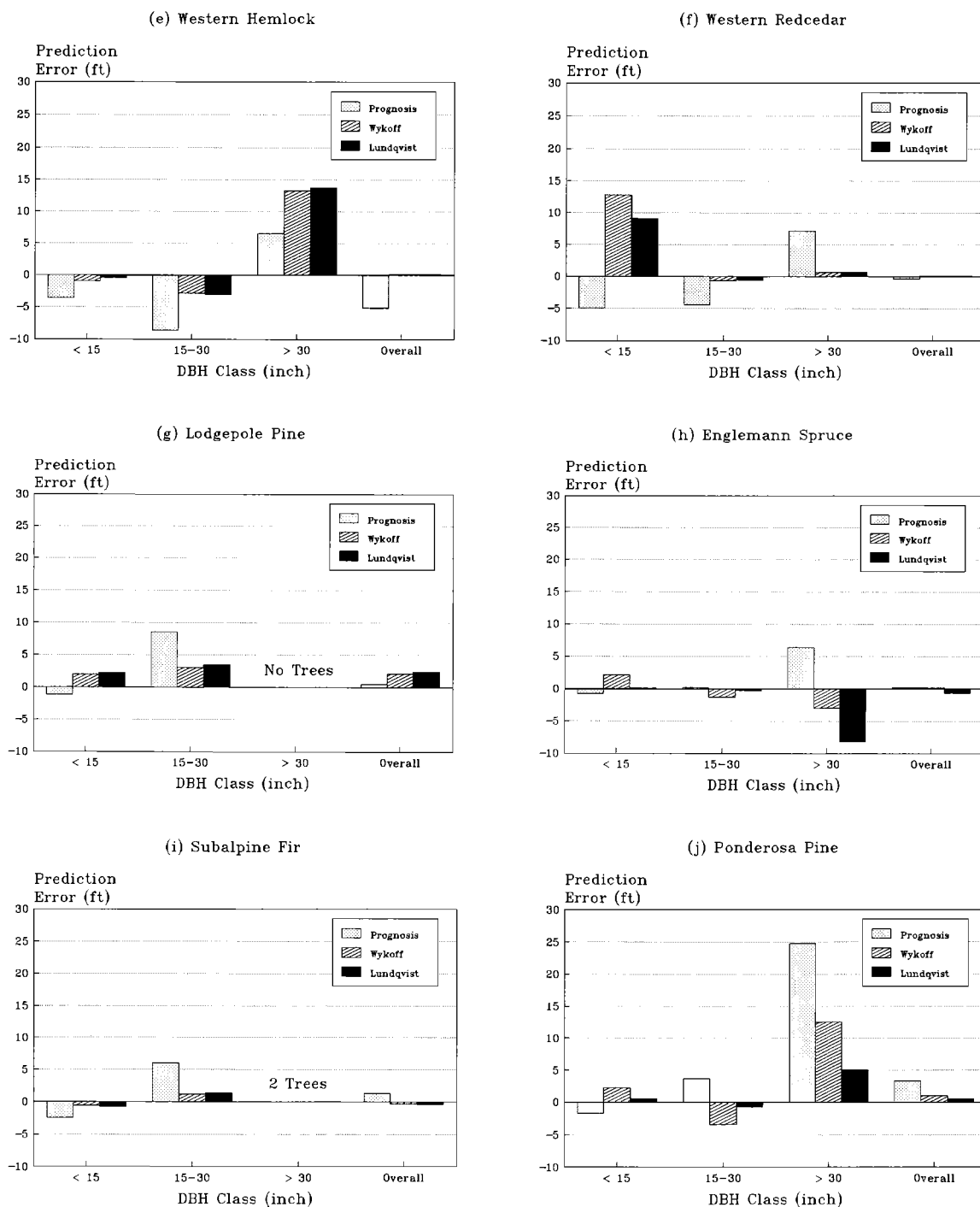(j) Ponderosa Pine

Prediction
Error (ft)

Figure 2 (cont.)

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